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Vacuum Wave Functional of Pure Yang-Mills Theory and Dimensional Reduction

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Abstract

Working in a Hamiltonian formulation with $A_0 = 0$ gauge and also in a path integral formulation, we show that the vacuum wave functional of four-dimensional pure Yang-Mills theory has the form of the exponential of a *three*-dimensional Yang-Mills action. This result implies that vacuum expectation values can be calculated in Yang-Mills theory but one dimension lower. Our analysis reveals that this dimensional reduction results from a stochastic nature of the theory.

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1 Introduction

In spite of considerable success in describing high energy phenomena of QCD, the dynamics at low energies, such as confinement and chiral symmetry breaking, has not satisfactorily been understood yet. Nonperturbative approaches are essential to study the low energy dynamics because of the strong coupling nature. One of successful nonperturbative approaches is the lattice formulation. The strong coupling expansion on the lattice has succeeded to show an area law for Wilson loops[1]. This strong coupling result seems to catch the essence of quark confinement but a dissatisfaction of the lattice formulation is that the connection with continuum field theory has been established only numerically. Furthermore, the lattice formulation encounters a notorious fermion doubling problem[2] and hence has much trouble in studying spontaneous chiral symmetry breaking. Thus, it will be important to develop nonperturbative methods in the continuum formulation to study the low energy dynamics of nonabelian gauge theories more transparently.

In this paper, we shall first examine the vacuum structure of pure Yang-Mills theory in the continuum Hamiltonian formulation, which will be well suited to study the nonperturbative dynamics of the theory. Our aim is then to solve the ground state of the Schrödinger wave functional equation

$$H\Psi_0[A] = E_0\Psi_0[A] \quad , \tag{1}$$

where H is the Yang-Mills Hamiltonian. An approximate vacuum wave functional in the infrared regime has originally been suggested by Greensite[3] to be of the form

$$\Psi_0[A] = \ N \ \exp\left\{-\frac{\gamma}{g^4}\int d^3x(F_{ij}^a)^2\right\} \quad , \tag{2}$$

where F_{ij}^a is a magnetic component of the field strength and γ is a numerical constant. This result has been supported by other studies: A lattice version of the wave functional (2) has been obtained in the strong coupling expansion of the lattice Hamiltonian formulation[4] and also studied in the Monte Carlo method[5]. The wave functional (2) has been rederived in the continuum strong coupling expansion by Mansfield[6]. The form of the wave functional (2) implies that vacuum expectation values can be calculated in a path integral representation of *three*-dimensional Yang-Mills theory. Three-dimensional gauge theories have been studied by various authors. Polyakov[7] has shown that three-dimensional compact QED has a mass gap and confines electric charges. For nonabelian gauge theories in three dimensions, Feynman[8] and recently many authors[9] have discussed the existence of a mass gap. Greensite has shown an area law of Wilson loops in an analog gas approximation in his original work[3]. The arguments of the $D = 4 \to D = 3$ dimensional reduction might work again for resulting three-dimensional Yang-Mills theory and then the theory would reduce to two-dimensional Yang-Mills theory, which exhibits confinement trivially[10, 6]. The vacuum wave functional (2) strongly suggests that in a strong coupling regime the vacuum consists of random magnetic fluxes, which has been discussed to be a necessary and sufficient condition for confinement[11, 12].

Although the wave functional (2) is a good candidate for the vacuum and possesses desired properties, all of the previous works have not verified that it is really the vacuum, i.e., the lowest energy state because they have looked for solutions of the Schrödinger equation (1) by taking appropriate ansatzs of the vacuum wave functional. In this paper, we shall show that the wave functional (2) is the lowest energy state in a more convincing way. Our approach reveals that the $D = 4 \to D = 3$ dimensional reduction results from a stochastic nature of the theory: We shall show that four-dimensional Yang-Mills theory in the infrared regime is equivalently described by the following stochastic system:

$$\frac{\partial A_i^a(x,t)}{\partial t} = -\frac{g^2}{2}\left.\frac{\delta S_{3YM}[A]}{\delta A_i^a(x)}\right|_{A_i(x)=A_i(x,t)} + \eta_i^a(x,t) \quad , \tag{3}$$

where η_i^a is a Gaussian white noise and S_{3YM} is a three-dimensional Yang-Mills action. In the equilibrium limit $t \to \infty$, this system has been shown to be equivalent to the quantum theory with the action S_{3YM}[13].

This paper is organized as follows: In Sec. 2, we solve a regularized version of the Schrödinger equation (1) and show that the vacuum wave functional takes to be of the form (2) in the limit of the cutoff $s \to 0$. In Sec. 3, Euclidean four-dimensional Yang-Mills theory can equivalently be described by the Langevin equation (3) in the limit $s \to 0$. Sec. 4 is devoted to conclusion.

2 Vacuum Wave Functional

We shall consider pure Yang-Mills theory whose Lagrangian is given by

$$\mathcal{L} = -\frac{1}{4g^2}F_{\mu\nu}^a F^{\mu\nu a} \quad , \tag{4}$$

where $F_{\mu\nu}^a$ is the field strength defined by

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc}A_\mu^b A_\nu^c \quad . \tag{5}$$

For our purposes, it is convenient to choose the $A_0 = 0$ gauge. In the Schrödinger representation, the (unregulated) Hamiltonian is then given by

$$H = \int d^3x \left\{ -\frac{g^2}{2}\frac{\delta^2}{\delta A_i^a(x)\delta A_i^a(x)} + \frac{1}{4g^2}\left(F_{ij}^a(x)\right)^2 \right\} \quad . \tag{6}$$

Here Latin indices i, j, k etc. run over the values 1,2, and 3. In the $A_0 = 0$ gauge, the wave functional $\Psi[A]$ has to be subject to the Gauss' law constraint

$$\left(D_i \frac{\delta}{\delta A_i(x)}\right)^a \Psi[A] = 0 \quad , \tag{7}$$

where D_i denotes a covariant derivative. This constraint simply means that the wave functional is invariant under time-independent gauge transformations. The

Schrödinger equation (1) with the Hamiltonian (6) needs regularization because it contains a product of two functional derivatives at the same spatial point.

$$\Delta \equiv \int d^3x \frac{\delta^2}{\delta A_i^a(x)\delta A_i^a(x)} \quad . \tag{8}$$

To make the differential operator (8) well defined, we replace Δ by the following differential operator[6]:

$$\Delta(s) \equiv \int d^3x d^3y \frac{\delta}{\delta A_i^a(x)} K_{ij}^{ab}(x,y;s) \frac{\delta}{\delta A_j^b(y)} \quad . \tag{9}$$

The kernel $K_{ij}^{ab}(x,y;s)$ is required to satisfy a heat equation

$$\frac{\partial}{\partial s} K_{ij}^{ab}(x,y;s) = \left[\delta_{ik}\left(D^2(x)\right)^{ac} - (D_i(x)D_k(x))^{ac} - 2f^{acd}F_{ik}^d(x)\right] K_{kj}^{cb}(x,y;s) \quad , \tag{10}$$

with the initial condition

$$\lim_{s\to 0} K_{ij}^{ab}(x,y;s) = \ \delta_{ij}\ \delta^{ab}\ \delta^3(x-y) \quad . \tag{11}$$

Taking s small but nonzero in Eq.(9) gives a regularized operator of Δ. (In the naive limit $s \to 0, \Delta(s)$ is reduced to Δ.) It should be emphasized that the regularized operator $\Delta(s)$ preserves gauge invariance and (three-dimensional) Lorentz invariance.

The heat equation (10) can be solved by the standard technique[14]. Acting $\Delta(s)$ on three-dimensional integrals of local functions will give an expansion in powers of s and may contain inverse powers of s, which diverge as $s \to 0$. These powers of s may be determined from dimensional analysis and gauge invariance. We have, for example,

$$\begin{aligned}
&\int d^3x d^3y \frac{\delta}{\delta A_i^a(x)} K_{ij}^{ab}(x,y;s) \frac{\delta}{\delta A_j^b(y)} \int d^3z (F_{kl}^c(z))^2 \\
&= \int d^3x \left\{ \frac{\alpha_1}{s^{5/2}} + \frac{\alpha_2}{s^{1/2}}(F_{ij}^a)^2 \right. \\
&\qquad \left. + s^{1/2}\left(\alpha_3 (D_i^{ab}F_{ij}^b)^2 + \alpha_4 f^{abc}F_{ij}^aF_{jk}^bF_{ki}^c\right) + \mathcal{O}(s^{3/2})\right\} \quad ,
\end{aligned} \tag{12}$$

where α_n's are numerical constants. The first two coefficients are given by

$$\alpha_1 = \frac{3 \dim G}{2\pi^{3/2}} \quad , \qquad \alpha_2 = -\frac{11 C_2(G)}{24\pi^{3/2}} \quad , \tag{13}$$

where $\dim G$ is the number of generators of the gauge group G and $C_2(G)$ is given by $f^{acd}f^{bcd} = C_2(G)\delta^{ab}$.

We now have a regularized Hamiltonian

$$H[A;s] = \int d^3x \left\{ -\frac{g^2}{2} \int d^3y \frac{\delta}{\delta A_i^a(x)} K_{ij}^{ab}(x,y;s) \frac{\delta}{\delta A_j^b(y)} + \frac{1}{4g^2}\left(F_{ij}^a(x)\right)^2 \right\} \quad . \tag{14}$$

Let us rewrite the regularized Hamiltonian (14) into the form

$$H[A;s] = \int d^3x d^3y \; Q_i^{a\dagger}(x) K_{ij}^{ab}(x,y;s) Q_j^b(y) + \Gamma[A;s] \quad . \tag{15}$$

The operators Q_i^a and $Q_i^{a\dagger}$ are defined by

$$\begin{aligned} Q_i^a(x) &= i\frac{g}{\sqrt{2}}\left(\frac{\delta}{\delta A_i^a(x)} + \frac{1}{2}\frac{\delta S_{3YM}[A]}{\delta A_i^a(x)}\right) \quad , \\ Q_i^{a\dagger}(x) &= i\frac{g}{\sqrt{2}}\left(\frac{\delta}{\delta A_i^a(x)} - \frac{1}{2}\frac{\delta S_{3YM}[A]}{\delta A_i^a(x)}\right) \quad , \end{aligned} \tag{16}$$

where

$$S_{3YM}[A] = \frac{24\pi^{3/2}s^{1/2}}{11C_2(G)g^4}\int d^3x \left(F_{ij}^a(x)\right)^2 \quad . \tag{17}$$

A key observation is that $\Gamma[A;s]$ in Eq.(15) vanishes in the naive limit $s \to 0$. It is easy to see from the formula (12) that $\Gamma[A;s]$ has the form

$$\Gamma[A;s] = \int d^3x \left\{ s(\beta_1(D_i^{ab}F_{ij}^b)^2 + \beta_2 f^{abc}F_{ij}^a F_{jk}^b F_{ki}^c) + \mathcal{O}(s^2) \right\} \quad , \tag{18}$$

up to a field independent constant. The $\Gamma[A;s]$ contains only higher dimensional terms with positive powers of s. Thus, in the naive limit $s \to 0$, $\Gamma[A;s]$ vanishes[4] (up to an irrelevant constant). Therefore, in the limit $s \to 0$, the Hamiltonian (14) may be replaced by

$$\overline{H}[A;s] \equiv \int d^3x d^3y \; Q_i^{a\dagger}(x) K_{ij}^{ab}(x,y;s) Q_j^b(y) \quad , \tag{19}$$

up to an irrelevant constant. Since the kernel K_{ij}^{ab} is positive definite, $\overline{H}$ is positive semi-definite. Thus, a zero energy eigenstate of $\overline{H}$, if any, is the lowest energy state, i.e., the vacuum[5],

$$\overline{H}\Psi_0[A] = 0 \quad . \tag{20}$$

It follows from the form (19) that the above equation is equivalent to solve

$$Q_i^a(x)\Psi_0[A] = 0 \quad , \tag{21}$$

which leads to a solution

$$\Psi_0[A] = N \; \exp\left\{-\frac{1}{2}S_{3YM}[A]\right\} \quad , \tag{22}$$

as announced in the introduction. It should be emphasized that we have not assumed any specific form of the vacuum wave functional to derive Eq.(22). Since S_{3YM} in

[4] In taking the limit $s \to 0$, we have to take the s dependence of the gauge coupling g into account because g in the Hamiltonian (14) is a bare coupling and should depend on the cutoff s for the theory to be renormalizable. We can show that $\Gamma[A;s]$ still vanishes as $s \to 0$ even if the s dependence of g is taken into account.

[5] Greensite[3] has found a zero energy solution to the *unregulated* Hamiltonian. The wave functional is not, however, normalizable and hence does not seem to have physical meaning.

Eq.(17) is positive semi-definite, the wave functional $\Psi_0[A]$ is normalizable, as it should be.

Let $F[A]$ be any functional of A_i^a. The vacuum expectation value of $F[A]$ can be expressed as

$$\int \mathcal{D}A_i \ \Psi_0^*[A]F[A]\Psi_0[A] = N^2 \int \mathcal{D}A_i \ F[A] \ \exp\{-S_{3YM}[A]\} \quad . \tag{23}$$

The last expression is identical to a path integral representation of three-dimensional Yang-Mills theory. Vacuum expectation values of any physical operators have to be independent of the cutoff s, so that the coupling constant g should be regarded as a function of s[6]. It follows from Eqs.(17) and (23) that the s dependence of g should be given by[3, 6]

$$g(s)^4 s^{-1/2} = s - \text{independent} \quad . \tag{24}$$

3 Stochastic Quantization Point of View

In the previous section, we have derived the vacuum wave functional (22) in the Hamiltonian formulation. In what follows, we shall show that the same conclusion (22) can be obtained from a stochastic quantization point of view[7].

Let us start with the following Langevin equation:

$$\frac{\partial A_i^a(x,t)}{\partial t} = -\frac{g^2}{2} \left. \frac{\delta S_{3YM}[A]}{\delta A_i^a(x)} \right|_{A_i(x)=A_i(x,t)} + \eta_i^a(x,t) \quad , \tag{25}$$

where η_i^a is a Gaussian white noise and $S_{3YM}[A]$ is given in Eq.(17). The average over η_i^a is defined by

$$\langle F[A^\eta] \rangle_\eta = N' \int \mathcal{D}\eta_i \ F[A^\eta] \ \exp\left\{ -\frac{1}{2g^2} \int d^3x dt (\eta_i^a(x,t))^2 \right\} \quad , \tag{26}$$

where F is an arbitrary function of A_i^a, N' is a normalization constant, and A^η exhibits the η dependence as a solution of the Langevin equation (25). We shall now show that the η average (26) can be rewritten as

$$\langle F[A^\eta] \rangle_\eta = N' \int \mathcal{D}A_\mu \ F[A] \ \delta(A_0) \ \exp\{-S_{4YM}[A]\} \quad , \tag{27}$$

where S_{4YM} is the (Euclidean) four-dimensional Yang-Mills action. The right hand side of Eq.(27) is nothing but a path integral representation of Euclidean four-dimensional Yang-Mills theory in the $A_0 = 0$ gauge. The equality in Eq.(27) should be understood in the same sense that H in Eq.(14) is replaced by $\overline{H}$ in Eq.(19) in the limit $s \to 0$. To show the relation (27), we will change the variables from η_i^a to A_i^a in

[6] In our field definition (5), there is no wave function renormalization.

[7] For reviews, see Ref. [15].

Eq.(26) through the equation (25). Then, the exponent of Eq.(26) can be rewritten as[8]

$$
\begin{aligned}
-\frac{1}{2g^2}\int d^3xdt(\eta_i^a(x,t))^2 &= -\frac{1}{2g^2}\int d^3xdt\,\left(\frac{\partial A_i^a(x,t)}{\partial t}+\frac{g^2}{2}\left.\frac{\delta S_{3YM}[A]}{\delta A_i^a(x)}\right|_{A_i(x)=A_i(x,t)}\right)^2 \\
&= -\frac{1}{2g^2}\int d^3xdt\,\left(\frac{\partial A_i^a(x,t)}{\partial t}\right)^2+\mathcal{O}(s)\quad, \qquad (28)
\end{aligned}
$$

where we have dropped a total derivative term in the last equality. We next calculate the Jacobian.

$$
\begin{aligned}
\det\left(\frac{\delta\eta_i^a}{\delta A_j^b}\right) &= \det\left(\frac{\partial}{\partial t}+\frac{g^2}{2}\frac{\delta^2 S_{3YM}[A]}{\delta A_i^a\delta A_j^b}\right) \\
&= \exp\left\{\int d^3xdt\,\frac{g^2}{4}\left.\frac{\delta^2 S_{3YM}[A]}{\delta A_i^a(x)\delta A_i^a(x)}\right|_{A_i(x)=A_i(x,t)}\right\}\quad, \qquad (29)
\end{aligned}
$$

where we have chosen the retarded Green's function of $\frac{\partial}{\partial t}$ to show the last equality in Eq.(29)[15], and omitted a field independent constant in Eq.(29). The expression on the right hand side of Eq.(29) is, however, ill defined because it contains a product of two functional derivatives at the same spatial point, as found in the Hamiltonian (6). According to the prescription discussed in the previous section, we will regularize the product of two functional derivatives. A regularized Jacobian is then given by

$$
\begin{aligned}
\det\left(\frac{\delta\eta_i^a}{\delta A_j^b}\right)_{reg} &= \exp\left\{\int d^3xd^3ydt\,\frac{g^2}{4}\left.\frac{\delta}{\delta A_i^a(x)}K_{ij}^{ab}(x,y;s)\frac{\delta}{\delta A_j^b(y)}S_{3YM}[A]\right|_{A_i(x)=A_i(x,t)}\right\} \\
&= \exp\left\{-\int d^3xdt\,\frac{1}{4g^2}\left(F_{ij}^a(x,t)\right)^2+\mathcal{O}(s)\right\}\quad, \qquad (30)
\end{aligned}
$$

where we have used the formula (12) and ignored an irrelevant constant. Combining Eqs. (28) and (30), we finally arrive at the conclusion (27), i.e.,

$$
\begin{aligned}
\langle F[A^\eta]\rangle_\eta &= N'\int\mathcal{D}A_i\ F[A]\ \det\left(\frac{\delta\eta}{\delta A}\right)_{reg}\exp\left\{-\frac{1}{2g^2}\int d^3xdt\left(\dot{A}_i^a+\frac{g^2}{2}\frac{\delta S_{3YM}}{\delta A_i^a}\right)^2\right\} \\
&\propto \int\mathcal{D}A_i\ F[A]\ \exp\left\{-\int d^3xdt\left\{\frac{1}{2g^2}(\dot{A}_i^a)^2+\frac{1}{4g^2}(F_{ij}^a)^2+\mathcal{O}(s)\right\}\right\} \\
&= \int\mathcal{D}A_\mu\ F[A]\ \delta(A_0)\ \exp\left\{-\int d^3xdt\left\{\frac{1}{4g^2}(F_{\mu\nu}^a)^2+\mathcal{O}(s)\right\}\right\}\quad. \qquad (31)
\end{aligned}
$$

Thus, we may conclude that four-dimensional Yang-Mills theory can equivalently described by the stochastic system governed by the Langevin equation (25). As seen

[8] In Eqs. (28), (30) and (31), we have not taken account of the s dependence of the gauge coupling g given in Eq. (24). Even if the s dependence of g has been taken account of, the leading terms shown in Eqs. (28), (30) and (31) are still correct.

above, t in the Langevin equation (25) corresponds to the Euclidean time, though t is usually regarded as a fictitious time in a stochastic quantization point of view[15].

In the operator language, the right hand side of Eq.(27) may be written as

$$N' \int_{A_\mu(T)=A_\mu(0)} \mathcal{D}A_\mu \; F[A] \; \delta(A_0) \; \exp\left\{ -\int d^3x \int_0^T dt \frac{1}{4g^2}(F^a_{\mu\nu})^2 \right\} = \mathrm{Tr}\left(F[A] \mathbf{e}^{-T\overline{H}} \right) \quad , \quad (32)$$

where we have chosen a periodic boundary condition, $A^a_\mu(T) = A^a_\mu(0)$. Taking the limit $T \to \infty$ gives

$$\begin{aligned} \lim_{T\to\infty} \mathrm{Tr}\left(F[A]\mathbf{e}^{-T\overline{H}} \right) &= \lim_{T\to\infty} \sum_n \mathbf{e}^{-TE_n} \langle n|F[A]|n\rangle \\ &\simeq \lim_{T\to\infty} \mathbf{e}^{-TE_0} \langle 0|F[A]|0\rangle \\ &= \int \mathcal{D}A_i \; \Psi_0^*[A] F[A] \Psi_0[A] \quad . \qquad (33) \end{aligned}$$

On the other hand, the Parisi-Wu dimensional reduction implies that the left hand side of Eq.(27) will be reduced to[13]

$$\begin{aligned} \lim_{T\to\infty} \langle F[A^\eta]\rangle_\eta &= \lim_{T\to\infty} N' \int \mathcal{D}\eta_i \; F[A^\eta] \; \exp\left\{ -\frac{1}{2g^2} \int d^3x \int_0^T dt (\eta^a_i)^2 \right\} \\ &= N' \int \mathcal{D}A_i \; F[A] \; \exp\left\{ -S_{3YM}[A] \right\} \quad . \qquad (34) \end{aligned}$$

Comparing Eq.(33) with Eq.(34), we arrive at the same vacuum wave functional (22) that we have derived in the Hamiltonian formulation. We therefore conclude that the vacuum wave functional (22) results from a stochastic nature of the theory. It is interesting to note that if we regard the Langevin equation (25) as a mapping of A^a_i to η^a_i, it is a kind of Nicolai mapping[9][18], which implies the existence of a hidden supersymmetry[19].

4 Conclusion

We have shown that the vacuum wave functional has the form (22) in the naive limit $s \to 0$. This does not, however, mean that the wave functional (22) is an exact expression for the vacuum, as discussed in Ref.[3]. We have dropped higher dimensional terms because they are proportional to positive powers of s and hence might vanish in the limit $s \to 0$. Taking the naive limit $s \to 0$ can, however, be dangerous because the scaling behavior in Eq.(24) is different from what one expects in the weak coupling regime. This situation seems to be similar to what one finds in the strong coupling expansion of the lattice gauge theory[1]. Although we believe that our results are qualitatively correct, it is important to show how our results connect with the weak coupling regime of the theory to make our analysis quantitative.

[9] Claudson and Halpern[16] have given different Nicolai maps for Yang-Mills theory in four dimensions, based on the Chern-Simons action, which have been explicitly checked to all orders by Bern and Chan[17]. Connections with our results are unclear.

Finally we would like to make a comment on dimensional reduction. A simple picture of confinement has previously been proposed[20, 12]: Random magnetic fluxes are dominating field configurations in a confining QCD vacuum and the theory exhibits the Parisi-Sourlas dimensional reduction of the type $D = 4 \rightarrow D = 2$[21] in the infrared regime. It is well known that two-dimensional QCD trivially confines. Numerical studies have supported this idea[22]. Our observation in the previous section may be a simple realization of the above idea by successively applying our analysis to get the $D = 4 \rightarrow D = 3 \rightarrow D = 2$ dimensional reduction, though what we found in this paper is not the Parisi-Sourlas type but the Parisi-Wu type of dimensional reduction[10]. It would be of great interest to investigate low energy dynamics of nonabelian gauge theories in a stochastic quantization point of view furthermore.

[10] Recently, Kalkkin and Niemi[23] have discussed the Parisi-Sourlas dimensional reduction in the instanton approximation. Connections with our results are unclear.

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